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A Photon Free Method to Solve Radiation Transport Equations

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September 6, 2006

Grannlibakken Transport Workshop
Tahoe, CA, United States
September 9, 2006 through September 16, 2006

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A Photon Free Method to Solve Radiation Transfer Equations

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Abstract

- The multi-group discrete-ordinate equations of radiation transfer is solved for the first time by Newton's method.
- It is a photon free method because the photon variables are elminated from the radiation equations to yield a $N_{group}XN_{direction}$ smaller but equivalent system of equations.
- The smaller set of equations can be solved more efficiently than the original set of equations.
- Newton's method is more stable than the Semi-implicit Linear method currently used by conventional radiation codes

The Backward Euler, Multi-group, Discrete-ordinate equations to be solved are

$$\left(\frac{1}{c\Delta t} I + \mu_d D + \sigma_g(T)\right)\psi_{g,d} = \sigma_g(T) B_g(T) + \hat{s}_{g,d} + \frac{\psi^n_{g,d}}{c\Delta t}$$

$$C_p(T)\frac{T - T^n}{\Delta t} = \sum_{g,d} w_d \sigma_g(T)\left(\psi_{g,d} - B_g(T)\right) + q$$

- D is a finite difference matrix
- T is the temperature
- $\psi_{g,d}$ is the intensity for group g and direction d
- $B_g(T)$ is the blackbody function
- $C_p(T)$ and $\sigma_g(T)$ is the heat capacity and cross section
- Our goal is solve this system, written as $G(\psi,T)=0$.
- # unknowns = size(ψ) + size(T) = (n_x X n_d X n_g) + n_x

Ψ is elminated from G(Ψ,T)= 0 to give the smaller F(T)=0; this improves efficiency.

- Solve the transport equation formally

$$\left(\frac{1}{c\Delta t} I + \mu_d D + \sigma_g(T)\right)\psi_{g,d} = \sigma_g(T) B_g(T) + \hat{s}_{g,d} + \frac{\psi^n_{g,d}}{c\Delta t}$$

- Plug ψ into the temperature equation to get

$$\boxed{F(T) \equiv C_p(T)\frac{T - T^n}{\Delta t} - \sum_{g,d} w_d \sigma_g(T)\left(H^{-1}_{g,d}\left(\sigma_g(T) B_g(T) + \hat{s}_{g,d} + \frac{\psi^n_{g,d}}{c\Delta t}\right) - B_g(T)\right) - q}$$

- # of unknowns = n_x

Conventional deterministic methods do not solve G(Ψ,T)=0, but solve a linear approximation.

- The Semi-implicit Linear approximation
  - Lags $C^n_p = C_p(T^n)$ and $\sigma^n_g = \sigma_g(T^n)$, and then
  - Linearizes, $B_g(T) = B^n_g + B'_g(T-T^n)$, to give

$$\left(\frac{1}{c\Delta t} I + \mu_d D + \sigma_g^n\right)\psi_{g,d} = \sigma_g^n\left(B_g^n + B_g'\left(T - T^n\right)\right) + \hat{s}_{g,d} + \frac{\psi_{g,d}^n}{c\Delta t}$$

$$C_p^n \frac{T - T^n}{\Delta t} = \sum_{g,d} w_d \sigma_g^n \left(\psi_{g,d} - B_g^n - B_g'\left(T - T^n\right)\right) + q$$

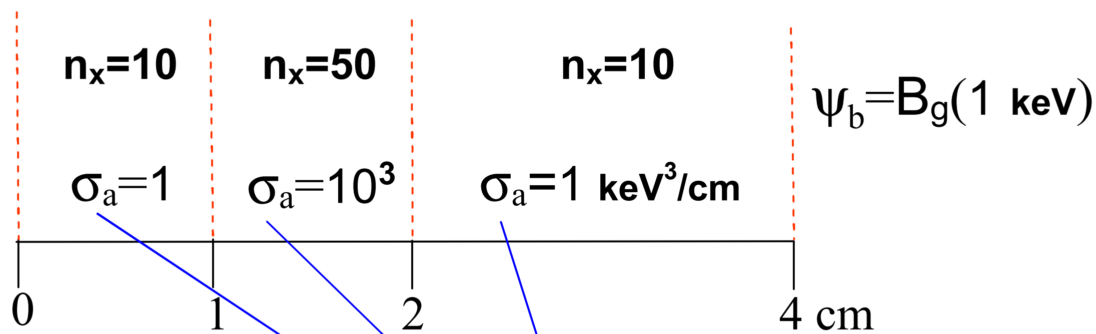
- Eliminating $(T-T^n)$ yields

$$\left(\frac{1}{c\Delta t} I + \mu_d D + \sigma_g^n\right)\psi_{g,d}^{SiL} = \tfrac{1}{2}\chi_g \eta \sum_{g',d'} w_{d'} \sigma_{g'}^n \psi_{g',d'}^{SiL} + \tilde{s}_{g,d}$$

may be negative; may yield negative ψ^{SiL} and T^{SiL}

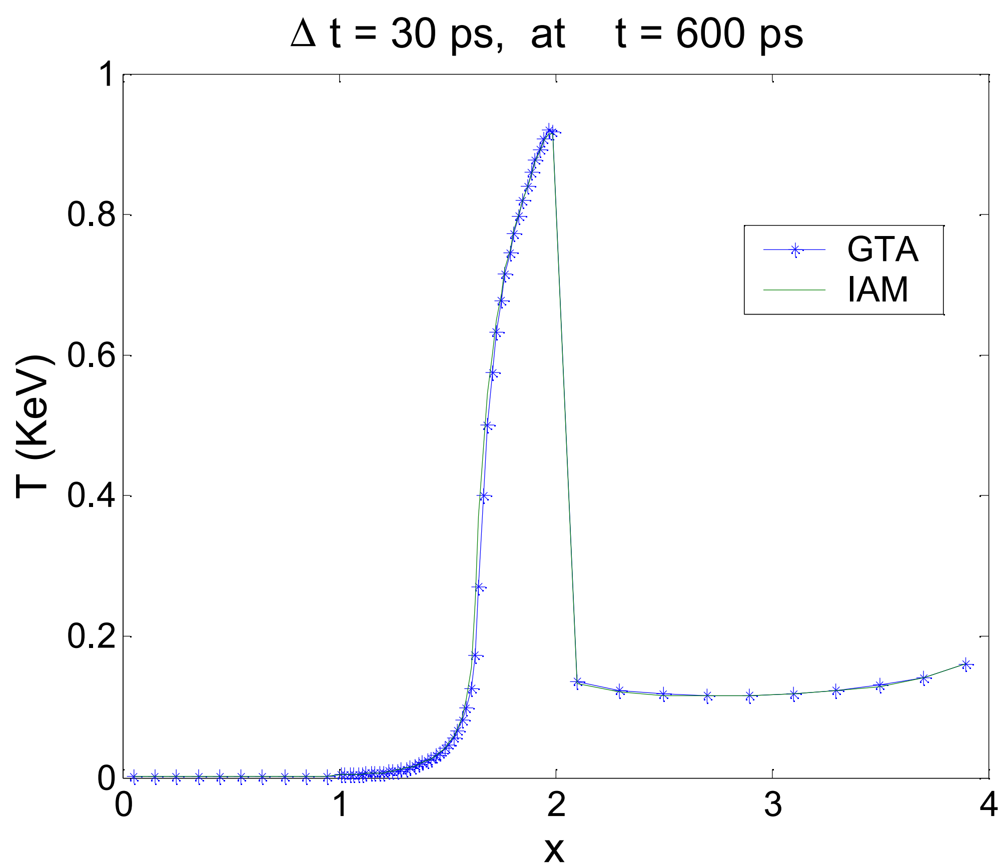
Test Problem

- $C_p = 5\text{x}10^{14}$ erg/cm^3-keV everywhere
- $N_d = 4$; $N_g = 50$, $h\nu_{min} = 10^{-5}$ keV, $h\nu_{max} = 10$ keV



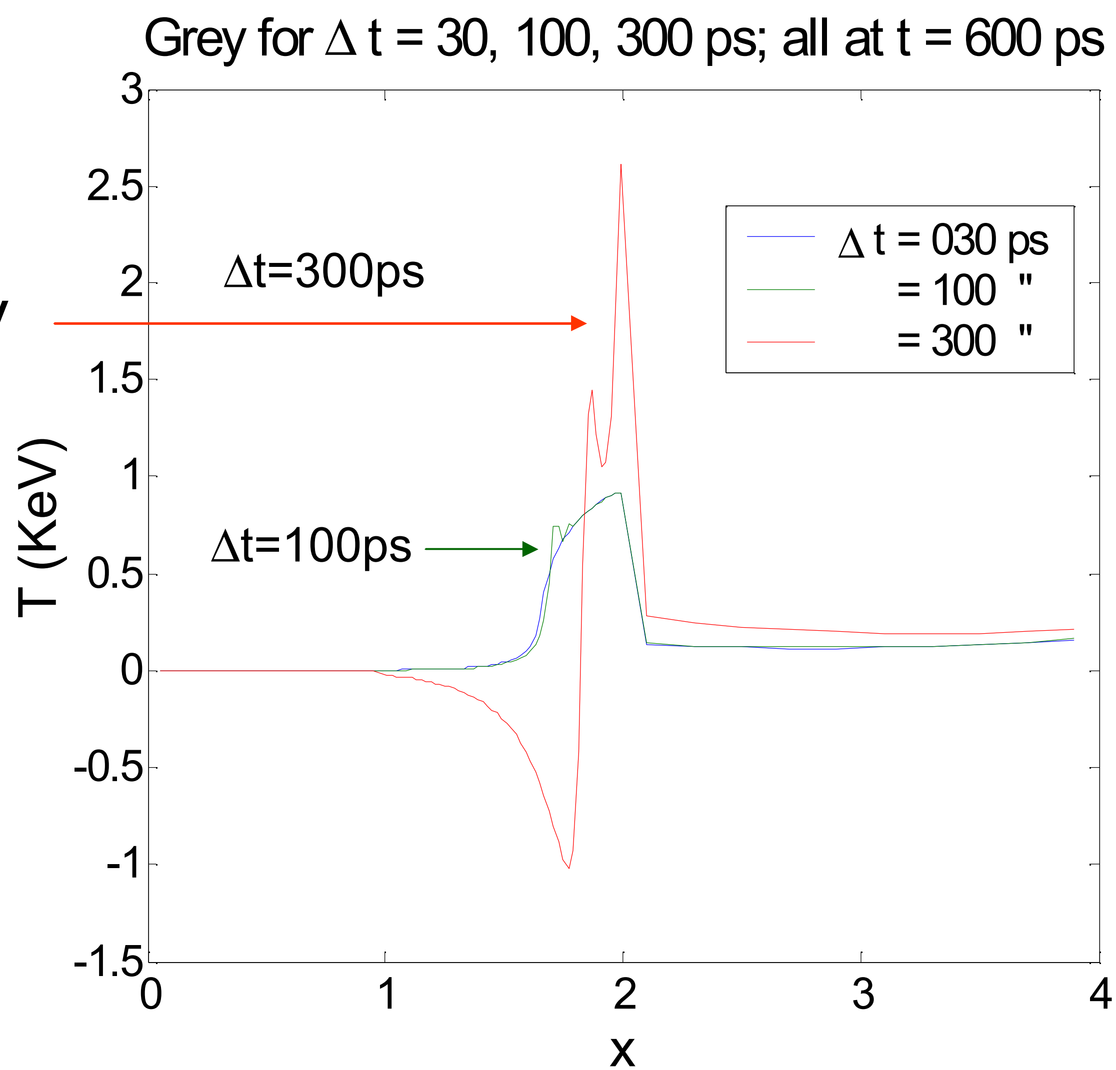
$$\sigma_\nu(T) = \sigma_a \frac{1 - e^{-h\nu/kT}}{\nu^3}$$

Linear vs. Nonlinear



Linear results for increasing Δt

Unstable: T > 1 keV



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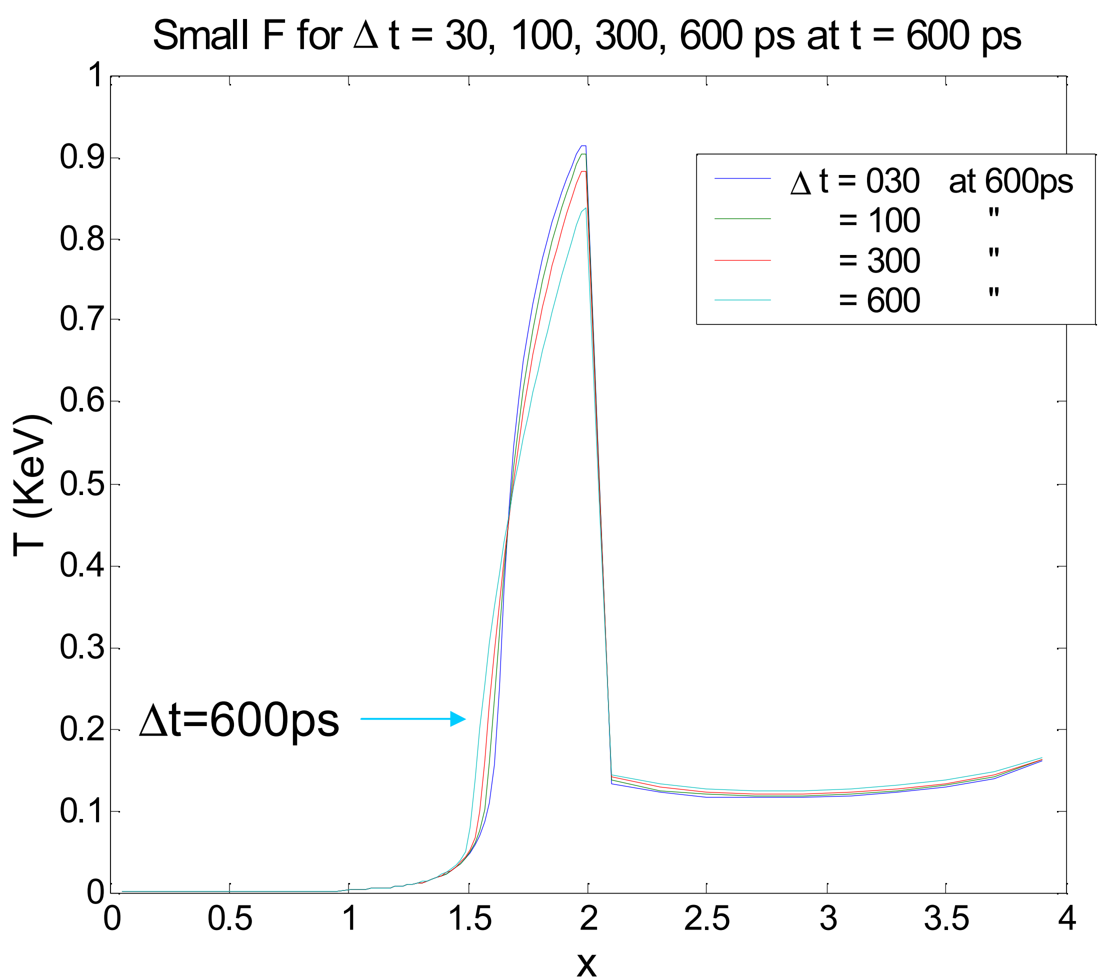
Nonlinear results for increasing Δt

Stable : T < 1 keV



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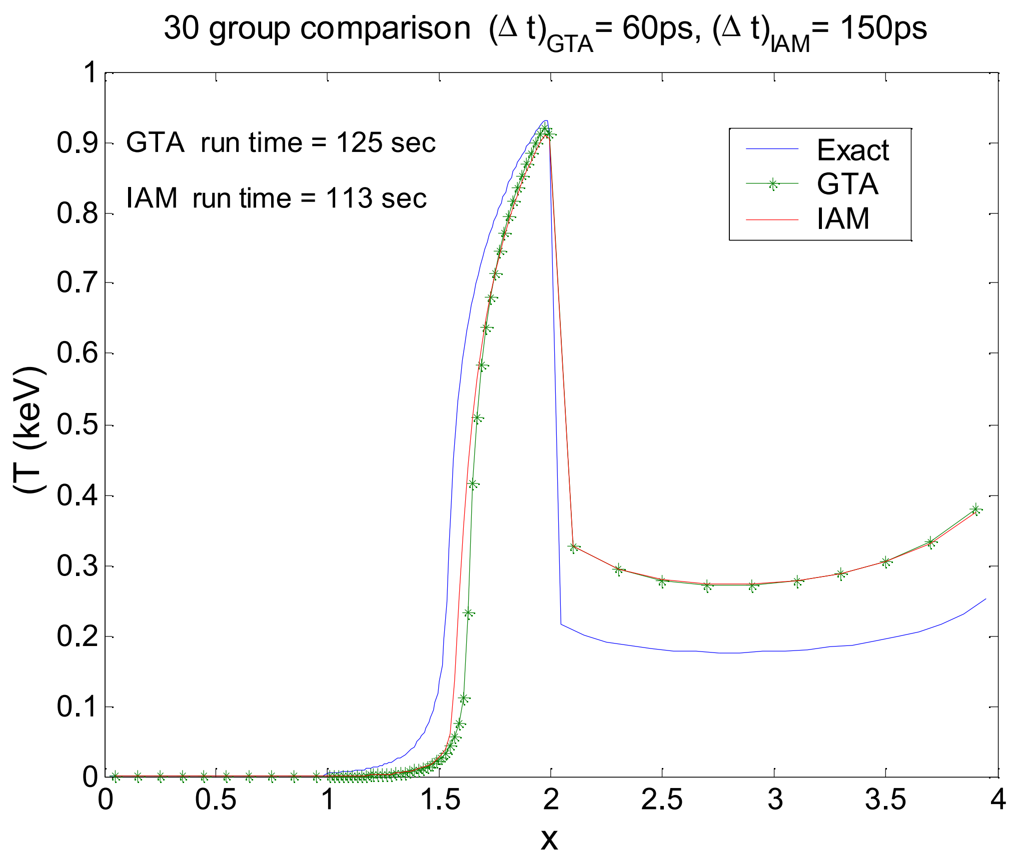
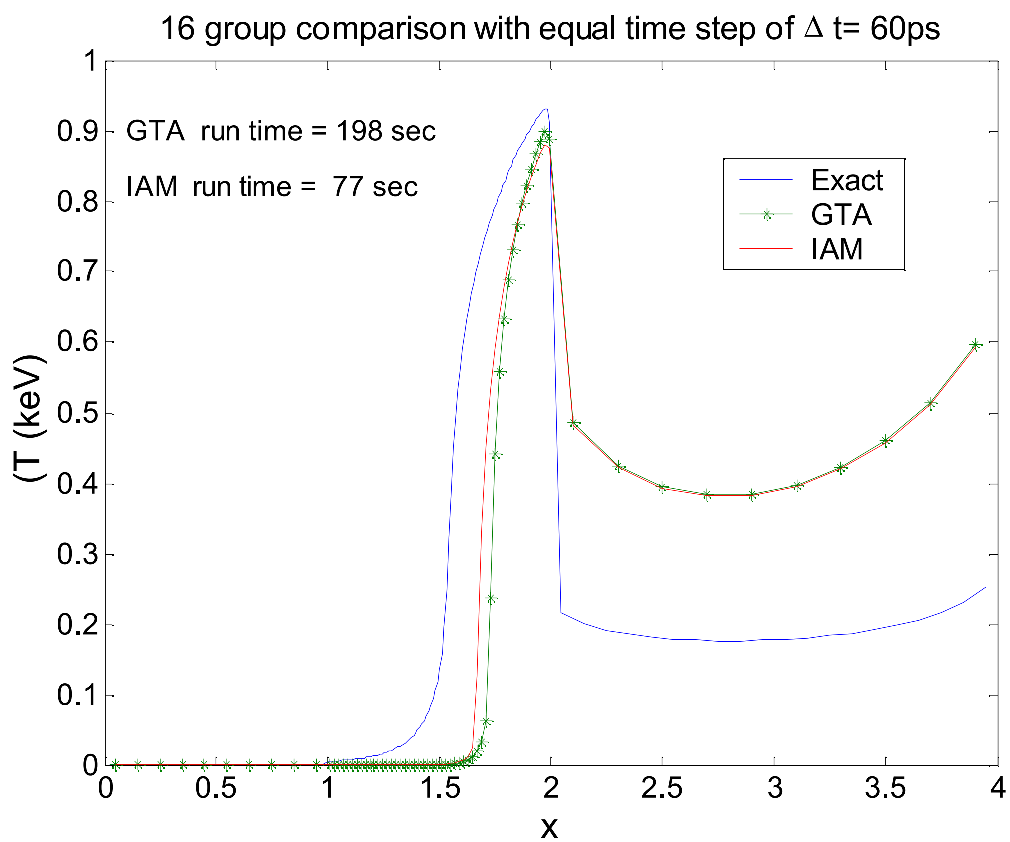
30 group comparison with different time steps

- Time step
  - Lin: 60 ps
  - NL: 150 ps
- Run time
  - Lin: 125 sec
  - NL: 113 sec



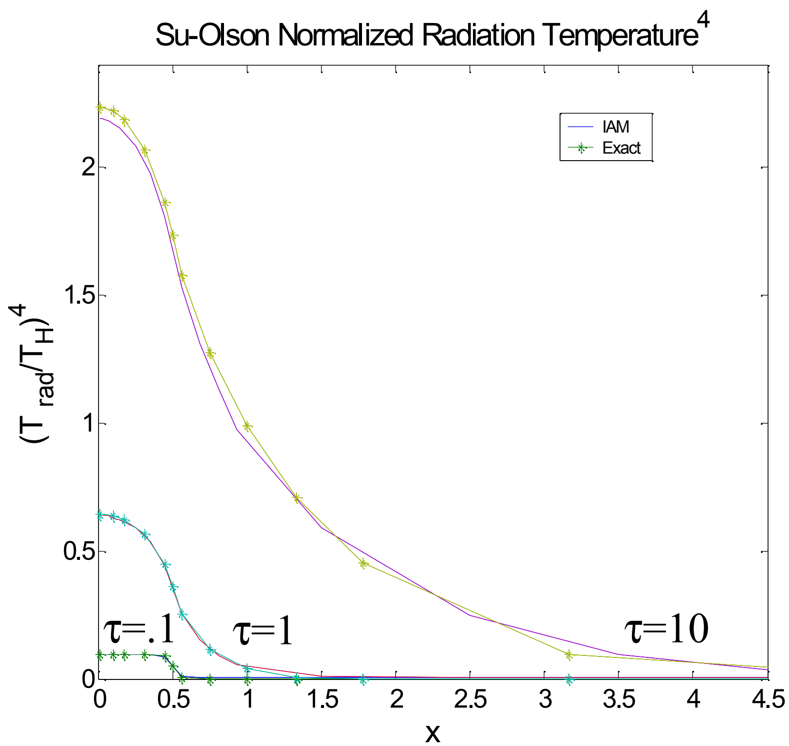
Nonlinear is as fast as linear

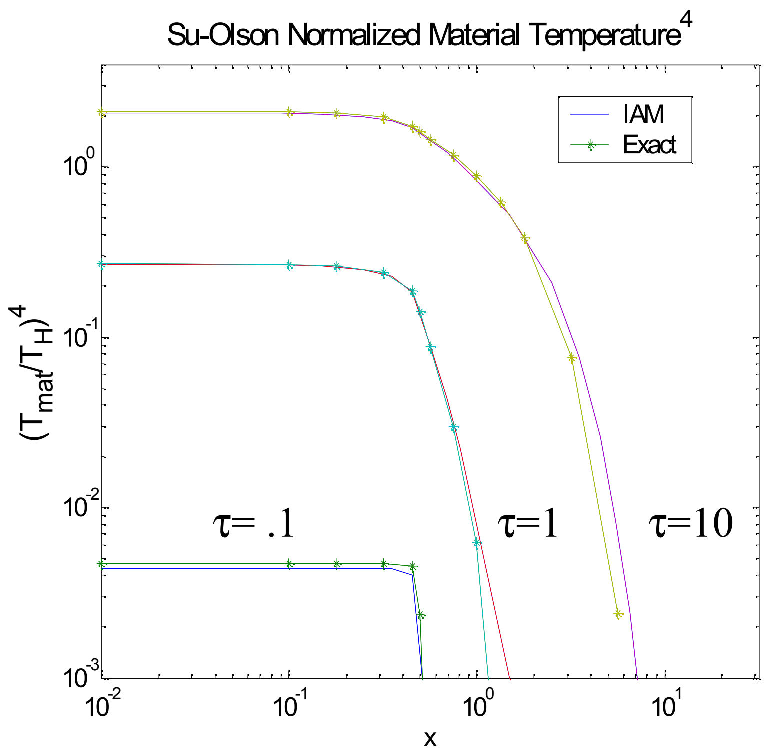
- Time step
  - Lin: 60 ps
  - NL: 60 ps
- Run time
  - Lin: 198 sec
  - NL: 77 sec



Nonlinear results for Su-Olson Problem

No scattering, Grey, $C_p = 4\, a_{rad} T^3$, $\sigma = 1\ \mathrm{cm}^{-1}$, $n_d = 20$, $n_x = 47$, $\Delta t = 334\mathrm{fs}$





Run time = 14 sec for $\tau = .1$

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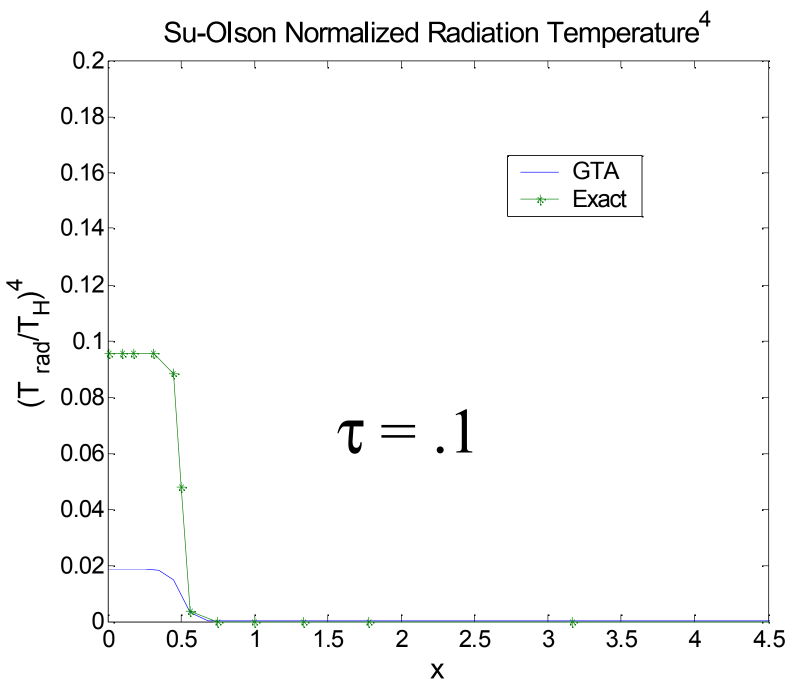
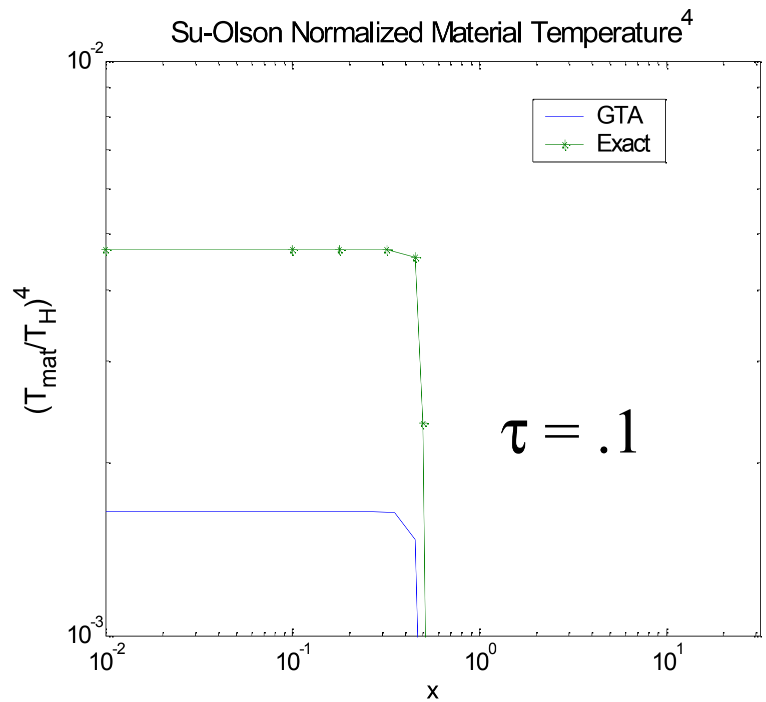
Linear results for Su-Olson Problem

No scattering, Grey, $C_p = 4\ a_{rad}T^3$, σ =1 cm^{-1}, n_d=20, n_x=47, Δt = .334fs





Run time = 1052 sec

Conclusion

- Newton's method is more stable than linear method
- Nonlinear method can be faster than linear method because the nonlinear method can take larger time steps than the linear method

This work was performed under the auspices of the U. S. Department of Energy by the University of California, Lawrence Livermore National Laboratory under Contract No. W-7405-Eng-48.